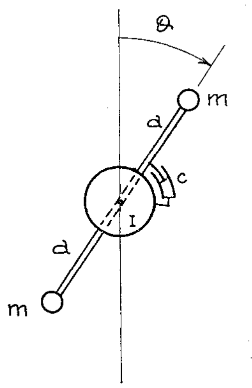

Fig. 1 Idealized dumbbell satellite with damper

this equation by 0, $-\alpha_1$, $-\alpha_2 \pm i\beta$. Forming the quartic that has these roots, one obtains

$$\lambda^4 + (\alpha_1 + 2\alpha_2)\lambda^3 + (\alpha_2{}^2 + 2\alpha_1\alpha_2 + \beta^2)\lambda^2 + \alpha_1(\alpha_2{}^2 + \beta^2)\lambda = 0 \quad (4)$$

If now one postulates light damping so that $\alpha_2{}^2$ and $\alpha_1\alpha_2$ are small as compared to β^2, Eq. (4) becomes

$$\lambda^4 + (\alpha_1 + 2\alpha_2)\lambda^3 + \beta^2\lambda^2 + \alpha_1\beta^2\lambda = 0 \quad (5)$$

Identifying the coefficients of Eqs. (3) and (4), one thus finds for the lightly damped case the four roots:

$$\lambda_1 = 0 \qquad \lambda_2 \doteq -(c/I)$$
$$\lambda_{3,4} \doteq -(c/4ma^2) \pm i3^{1/2}\Omega \quad (6)$$

The libration mode occurring at circular frequency $3^{1/2}\Omega$ thus appears to be quite well damped. The zero root in Eq. (6) arises because the flywheel is indifferent as to its rest position.

Reference

[1] Paul, B., "Planar librations of an extensible dumbbell satellite," AIAA J. 1, 411–418 (1963).

Equations for Specifying Orientation of a Planet-Orbiting Body for Yaw, Pitch, and Roll

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Recent developments in satellite design require more detailed analyses of the effects of solar heating on the vehicle—particularly with respect to complex motions, where yaw, pitch, and roll are significant.

Katz,[1] in his paper on solar heating, developed equations describing the orientation of an orbiting body referenced to its orbital plane; however, this paper did not contain equations that would describe the orientation of a spinning body in orbit which has been yawed and pitched or subjected to other complex motions. This paper contains equations that will permit consideration of a spinning, yawed, and pitched body.

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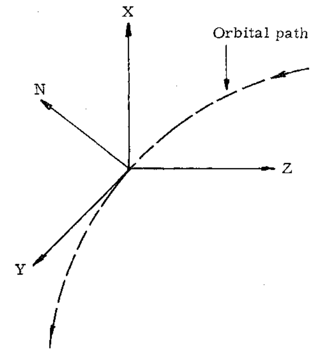

Fig. 1 Orientation of *X*, *Y*, and *Z* axes to orbital plane; *X* lies in orbital plane, is normal to orbital path, and points outward from orbital path; *Y* lies in orbital plane, is tangent to orbital path, and points in direction of body travel; *Z* forms a right-hand vector system with *X* and *Y*; *N* is a surface normal to any element area at body surface

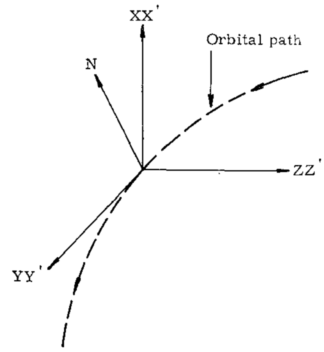

Fig. 2 *X′*, *Y′*, and *Z′* body axes coincident with *X*, *Y*, and *Z* axes at perigee, with no yaw, pitch, or spin

The general case of motion for an orbiting body, of course, would consider not only yaw, pitch, and spin but also periodic (oscillatory) motions, with and without damping. However, periodic damped or undamped motion will not be considered here, except insofar as spin may be thought of as a continuous periodic roll.

For convenience, definitions of yaw and pitch will be taken as fixed angular displacements about particular axes of rotation; spin will be taken as continuous body roll about a particular axis. In general, the definitions, as given below, will follow standard aerodynamic practice. Orientation of the unprimed X,Y,Z axes with respect to the orbital plane is shown in Fig. 1.

For the case in which an orbiting body is not yawed, pitched, spun, or subjected to other motions (Fig. 2), the orientation of any body element surface area normal is specified with respect to the X,Y,Z axes at perigee, by the angles α,β, and γ, respectively. However, when a body is subjected to motions such as yaw, pitch, and spin, the angles α,β, and γ may change by a fixed amount or continuously, such that some means must be devised to describe the orientation of body element surface area normals at any instant of time.

Yaw will be considered as a fixed angular displacement, always taken about the X axis (Fig. 3). Pitch will be considered as a fixed angular displacement about the Z' axis. This axis is referenced to a coordinate system, which is fixed with respect to the orbiting body (Fig. 3). Spin is taken as a continuous body roll about the Y' axis, which is referenced to the rotating body coordinate system (Fig. 3). The same assumptions are made here as are made in Ref. 1, with respect

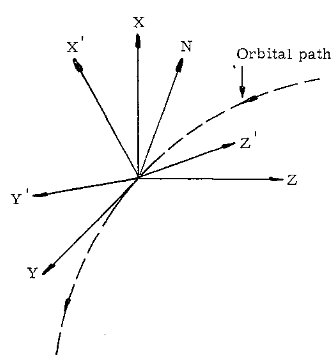

Fig. 3 *X′*, *Y′*, and *Z′* axes noncoincident with *X*, *Y*, and *Z* axes, with yaw, pitch, and spin

to elements of body area, to the specification of heat fluxes, and to surface normals.

Let $X'Y'Z'$ be a coordinate system fixed with respect to an orbiting body, so that, at perigee and with no displacements, X' coincides with X, Y' with Y, and Z' with Z. Then, the angles α,β, and γ define the orientation of any element surface area normal with respect to both primed and unprimed axes. When such a body is displaced or rotating in its orbital path, the orientation of an element surface area normal remains fixed with respect to the X',Y',Z' axes, and changes in the same manner as these axes change with respect to the X,Y,Z axes. Thus, by specifying the change between the primed and unprimed axes, one also specifies the change in orientation of an element surface area normal with respect to the unprimed coordinate system.

The orientation of the primed axes to the unprimed axes at any instant of time can be written with vector notation† in the general form

$$\alpha_d = \arc\cos[\cos\alpha_0 l_{XX'} + \cos\beta_0 l_{XY'} + \cos\gamma_0 l_{XZ'}] \quad (1)$$

$$\beta_d = \arc\cos[\cos\alpha_0 l_{YX'} + \cos\beta_0 l_{YY'} + \cos\gamma_0 l_{YZ'}] \quad (2)$$

$$\gamma_d = \arc\cos[\cos\alpha_0 l_{ZX'} + \cos\beta_0 l_{ZY'} + \cos\gamma_0 l_{ZZ'}] \quad (3)$$

where subscript 0 refers to the angles α,β, and γ, defined at perigee with respect to the unprimed axes, in the case in which no yaw, pitch, or roll occurs. Subscript d refers to the angles α,β, and γ (referenced to the unprimed axes) when these angles change because of body rotation. $l_{XX'}$, $l_{YY'}$, $l_{ZZ'}$, etc., are the dot products $\bar{\imath}\cdot\bar{\imath}'$,$\bar{\jmath}\cdot\bar{\jmath}'$, $\bar{k}\cdot\bar{k}'$, etc. More specifically, $l_{XX'}$, $l_{YY'}$, etc., are the direction cosines between the primed and unprimed axes.

To be directly applicable to specific problems, it is necessary to evaluate the expressions $l_{XX'}$,$l_{YY'}$, etc., as functions of yaw, pitch, and continuous body roll (spin). Since the resultant body position may not be always independent of the order in which displacements are made, it is necessary to specify a convention making any resultant body position always independent of any order of displacement. This is as follows:

1) Yaw, represented by the angle ρ, always is taken about the unprimed X axis.

2) Pitch, represented by the angle K, always is taken about the Z' axis; the Z' axis forms a right-hand system with the X' and Y' axes. The Z' axis is coincident with the Z axis at perigee, when the body has no yaw, pitch, or roll.

3) Spin, represented by the angle W, always is taken about the Y' axis; the Y' axis is the longitudinal body axis and is coincident with the Y axis at perigee, with no yaw, pitch, or roll.

4) The angle W is the total angle rolled through in a period of time, t sec, i.e., $W = W_t \times t$, where W_t = angular spin rate in degrees per second and t = time in seconds.

5) All rotations about axes follow the standard right-hand-advancing-screw convention, except for pitch, which is taken in a sense opposite to that convention.

Following the foregoing convention, Eqs. (1–3) now may be written in a form applicable to any specific problem. The notation is the same as was used previously:

$$\alpha_d = \arc\cos[\cos\alpha_0 \cos K \cos W + \cos\beta_0 \sin K + \cos\gamma_0 \cos K \sin W] \quad (4)$$

$$\beta_d = \arc\cos[-\cos\alpha_0(\cos\rho \sin K \cos W - \sin\rho \sin W) + \cos\beta_0 \cos\rho \cos K - \cos\gamma_0(\sin\rho \cos W + \cos\rho \sin K \sin W)] \quad (5)$$

$$\gamma_d = \arc\cos[-\cos\alpha_0(\sin\rho \sin K \cos W + \cos\rho \sin W) + \cos\beta_0 \sin\rho \cos K + \cos\gamma_0(\cos\rho \cos W - \sin\rho \sin K \sin W)] \quad (6)$$

† Equations (1–3) are taken from p. 154 of Ref. 2.

References

[1] Katz, A. J., "Determination of thermal radiation incident upon the surfaces of an earth satellite in an elliptical orbit," Inst. Aerospace Sci. Paper 60-58 (1958).

[2] Housner, G. W. and Hudson, D. E., *Applied Mechanics—Dynamics* (D. Van Nostrand Co. Inc., Princeton, N. J., 1958), p. 154.

Expanding Flow Ionization Nonequilibrium: Its Contribution to MHD Generators

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Nomenclature

A = nozzle cross-section area
T = temperature, °K
l = nozzle throat radius divided by $\tan\theta$
u = velocity
x = axial distance from nozzle throat toward nozzle exit
α = rate coefficient for cesium atoms and electrons, cm³/molecule-sec
γ = electron density divided by gas density
ρ = gas density
θ = half-angle of the asymptotic cone of the nozzle

Subscripts

eq = at equilibrium
T = at nozzle throat
x = at position x in nozzle

Eschenroeder[1] has surmised that significant performance gain for MHD generators can result from the nonequilibrium behavior of a seeding material, cesium ion, in expanding flow. Effectively, potential energy might be converted to kinetic energy without a large loss in ion concentration and conductivity before the fluid enters the generator. Shorter generators would be possible, and the shortened length itself would tend to minimize concentration relaxation.

The purpose of the following is to report a particular application of the foregoing surmise to present-day MHD generators in order to determine whether the nonequilibrium conductivity can be large enough to require inclusion in generator power calculations. The finding is that, in regimes of flow applicable to MHD generators, contribution of nonequilibrium ionization can be neglected.

Conditions of temperature and pressure purposely were chosen which are somewhat too severe for present-day continuous MHD generator operation and therefore are among the most conducive to levels of ionization nonequilibrium which may affect generator performance. These combustor and nozzle entrance conditions are stoichiometric combustion of propane/oxygen at 30 atm and 3560°K. The combustion products, including cesium seed material, undergo isentropic expansion through an axisymmetric, hyperbolic nozzle down to 2 atm static pressure at the MHD generator entrance.

A mathematical development of ionization nonequilibrium of cesium in expanding flow of combustion products is given by Eschenroeder.[1] It finds expression as a nonlinear differential equation for one-dimensional steady flow:

$$d\gamma/dx = -(\gamma^2 - \gamma_{eq}^2)(\rho\alpha/u)$$

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